WWWを用いた数学個別学習教材の開発
(統計的推測編)

Mathematics Software for Education on the World Wide Web
(A package for statistical inference)

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ABSTRACT. I made "After school math" software for high school mathematics, and published it on WWW 6 years ago. Since then many students of our high school have studied math with this software. Those who have made the best use of it were physically handicapped or students who could not come to school because of various reasons. These days many people outside our school are beginning to use this software. My software has an interactive interface which lets you enjoy the reaction of the machine and software.

概要 WWWを用いた視覚による統計教材ソフトの開発とその評価について６年前より，インターネット上で数学を視覚的に，インタラクティブに捉えることを基本概念に持つWWW上の数学個別学習支援教材「放課後の数学」を，全国の高校生はもちろんのこと，中学生，大学生，一般社会人を対象に提供している。今回はその中で，「数学Ｃ」という分野から，高校レベルという枠を取り外し，統計一般分野にまで範囲を広げ製作した。その特徴は，身近な生活における現象を，シュミレーションを行うことにより視覚的に捉えていこうとした。また，一方で，統計分野において「扱う個数が十分大」という言葉が用いられるが，どれくらいなのか見当がつかない。それも，グラフを用いることにより視覚的に確認ができるソフトを製作した。数学Ｃの分野は，ＩＰＡの支援を受け製作されている.

１．Introduction

Six years ago, I developed "After School Math" for the World Wide Web. I have worked hard to make the content along the lines of real life. But at first, I received a lot of negative opinions that initially math students should study math with a pencil and paper, because they could not develop their abilities with media materials.

However, the computer facilities at Kwansei Gakuin Hihg School are a good match for the students' abilities and skill. So I didn't need to worry. Furthermore electronic-mail from many people who used the software supported my developing the product.

The most important purpose of this software is to allow students to get mathematical images. Thus through the computer it's possible for all students to view images we (math teachers) or students can make. Computers can make images easier and more correct than we show on the blackboard.

On the other hand, we make students practice till they understand definitions, for example the definition of the trigonometric ratios or logarithms. While practicing on the computer, students are given many similar exercises repeatedly to reinforce the mathematical definitions. To reinforce classroom instruction the use of "After School Math" helps students study many different exercises produced at random by the software. Thus the software presents an individual learning program on WWW.

Now I'll introduce only the content of statistical distributions and inference fields in the software. Statistical fields are easy computations for a computer, because they can produce random numbers and do huge calculations.

At present, this software "After School Math" is free to the public and can be found at : URL: http://www.kwansei.ac.jp/hs/z90010/hyousi/2106.htm

２．The application for density functions

"After School Math" has two features in the statistical field.

(1) The visualization of graphs
(2) The simulation of statistical phenomena in the real

world

(1) The visualization of graphs

The first feature includes the following two characteristics, interactivity and visualization.

1-a. Interactivity

Studying statistics, we might like to know the shape of a density function, and how it changes if we change some of its parameters. By means of varying the value of parameters using handles, we can see the varying expressions and graphs visually in detail. Figure 1. illustrates a density function. By moving the left two handles with a mouse, the graph and expressions vary.

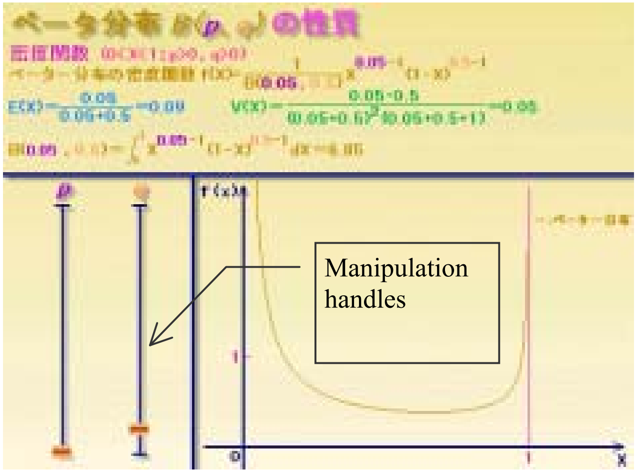

Figure 1. a density function

1-b. Visualization

The passage "as the 'n' is a large number" is often used in statistic textbooks, however, we don't know what number is large enough to make the graphs converge, i.e. we do not know the value of 'n'. By means of using "After School Math", it's possible for us to visualize how the graphs converge as we increase the parameters number. This tells us whether the number is large enough or not. Figure 2. shows that as the number 'n' (sample number) increases, the t-density function is converging to the normal density function. When the sample number is about 25, the t-density function conforms to the normal density function. In After School Math, the purpose is for students to gain a deep understanding by handling the parameter bar.

Since the content is somewhat difficult for high school students, I've been trying to develop the statistical teaching materials which are more concrete and based on the real world. Thus I paid attention to give it a great appeal to students.

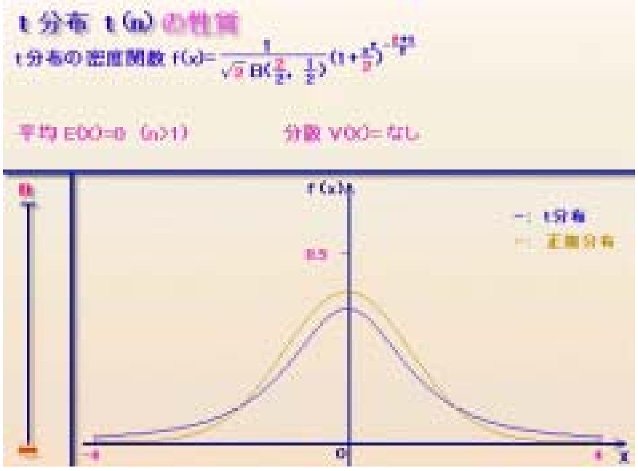

Figure 2. t- and normal density function

(2) The simulations of statistical phenomena in the real world

Here I'll introduce four statistical inferences, (a) the inference of the average on a finite population, (b) the inference of an area and the number π, (c) The Buffon needle problem, (d) The inference of the number of fish in a lake.

Figure 3. The inference of the average of a finite population

2-a The inference for a finite population average

It's from the simulation that we try to infer the average of a finite population which is produced on computer from the average of a sample. Originally we do not know the exact average of a population, since it is infinite or huge. Thus after we extract some elements from the population and calculate the average of the sample, we then infer the exact average of the population. Figure 3. shows this process. Now I will explain the handling of the data. The procedure is as follows

Step 1., the computer generates a population consisting of 1023 numbers.

Step 2., the student inputs the sample number into the textbox.

Step 3., the student pushes the shuffle button, then the sample numbers turn to red and the sample average number is shown.

Finally in Step 4., the student pushes the true population average which gives its value shown, at the button.

If the student pushes the button labeled “Shuffle” over and over again, the population of the estimated average will change. It’s the aim of this software to give them doubts about what number is adequate. Theoretically we should extract 150 elements, if we demand a high accuracy (the probability that the sample average is included in ±1 from the popular average of 99 %).

<u>2-b The inference of an area and the number π</u>

Figure 4. The inference of an area and the number π

This is a simulation by means of the well known Monte Carlo method. We are able to calculate any area by it, when we have the section in a given area on the condition that the area is known.

Step 1. By pushing first button, we decide on the section.

Step 2. By pushing the second button, in that area random points are generated by the computer, then we let the computer make the judgment of whether every point is included or excluded in the specified section.

Step 3. We are able to infer the area of the section from the ratio of included points to the number of excluded points in the given sections, A or B.

At first, students decide on the section’s shape and input the number of points, into the textbox. Figure 4. demands the section between the cubic curve and the x axis, where x=4.

<u>2-c The Buffon needle problem</u>

The next inference deals with the famous Buffon Needle Problem. After drawing many parallel lines on a plane, drop several needles on it. Then count the number of needles which cross the parallel lines. From this we are able to infer the number π from the ratio of the number of needles that crossed parallel lines to the overall total nuber of needles. The procedure is as follows.

Step 1. Input the length of the interval and trials’ number.

Step 2. The computer produce needles on the area as soon as push the first button is pushed.

Theoretically we are able to calculate the probability of a needle crossing a line, which includes the number π. On the one hand we get it by means of the simulation. Therefore we can generate the number π with the computer. The following Figure 5. shows the computer producing needles.

Figure 5. Showing the computer producing needles

<u>2-d The inference of the number of fish in a lake</u>

We heard that the number of fish were increasing or decreasing this year. But we don’t know how to calculate this variable in detail. Well, we let the computer produce an imaginary lake and 8,000-20,000 fish, we’ll infer the number of fish by the maximum likelihood method. However, we don’t know the actual number of fish in the lake, but it’s possible to compare the estimated number with the true number.

Following are the procedures for calculating the estimated number of fish in the lake.

Step 1. Input the number of fish which are caught in the imaginary lake. Mark the fish and put them back into the imaginary lake.

Step 2. Input the number of fish are in the second collection of fish. Then count how many marked fishes in it.

Step 3. By means of this method, we can infer the number from the values in Step 1.and Step 2.

Step 4. By pushing the maximum likelihood button (MLB), we can see and compare the inference value with the true value.

We recognize that the value generated by this method is not so correct. See Figure 6.

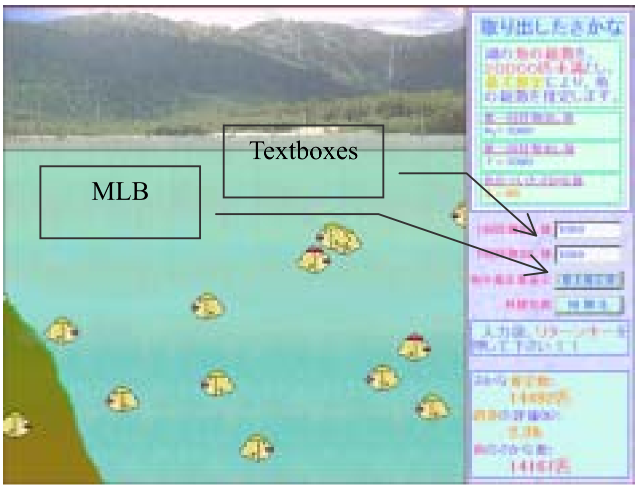

Figure 6. The inference of the fishes’ number

3．Remaining problems in the future

I can understand the theory from reading a textbook, but I don't know if its really true unless I can actually do the practice exercise myself. This is the purpose of "After School Math", to help students actually see how statistics operate in a real way. On the other hand, I plan to study the quantitative measurement of the educational effect when using media.

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